Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Matrices Practice

Morning (Time: 1 hour 30 minutes)

Paper reference **WFM03/01**

Mathematics

International Advanced Subsidiary/Advanced Level

Further Pure Mathematics F3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

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2. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

$$\mathbf{M} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 4 \\ 3 & -2 & -3 \end{pmatrix}$$

(a) Determine $\mathbf{M}^{-1}$

(3)

The transformation represented by **M** maps the plane Π_1 to the plane Π_2

The point (x, y, z) on Π_1 maps to the point (u, v, w) on Π_2

(b) Determine x, y and z in terms of u, v and w as appropriate.

(3)

The plane Π_1 has equation

$$3x - 7y + 2z = -3$$

(c) Find a Cartesian equation for Π_2

Give your answer in the form $au + bv + cw = d$ where a, b, c and d are integers to be determined.

(2)

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Question 2 continued

5.

$$\mathbf{M} = \begin{pmatrix} 1 & 2 & k \\ -1 & -3 & 4 \\ 2 & 6 & -8 \end{pmatrix} \quad \text{where } k \text{ is a constant}$$

Given that **M** has a repeated eigenvalue, determine

(i) the possible values of k,

(ii) all corresponding eigenvalues of **M** for each value of k.

(7)

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Question 5 continued

5.

$$\mathbf{A} = \begin{pmatrix} a & a & 1 \\ -a & 4 & 0 \\ 4 & a & 5 \end{pmatrix} \quad \text{where } a \text{ is a positive constant}$$

Given that 2 is an eigenvalue of **A**

(a) determine

(i) the value of a

(ii) the other two eigenvalues of **A**

(5)

A normalised eigenvector for the eigenvalue 2 is $\begin{pmatrix} \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \\ -\frac{2}{\sqrt{6}} \end{pmatrix}$

(b) Determine a normalised eigenvector for each of the other eigenvalues of **A**

(5)

(c) Hence determine a matrix **P** and a diagonal matrix **D** such that $\mathbf{P}^{\mathrm{T}}\mathbf{AP} = \mathbf{D}$

(2)

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Question 5 continued

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3.

$$\mathbf{A} = \begin{pmatrix} 2 & k & 2 \\ 2 & 2 & k \\ 1 & 2 & 2 \end{pmatrix} \qquad \text{where } k \text{ is a constant}$$

(a) Determine the values of k for which $\mathbf{A}$ is singular.

(2)

Given that $\mathbf{A}$ is non-singular,

(b) find $\mathbf{A}^{-1}$, giving your answer in terms of k.

(4)

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Question 3 continued

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5.

$$\mathbf{M} = \begin{pmatrix} 6 & -2 & -1 \\ -2 & 6 & -1 \\ -1 & -1 & 5 \end{pmatrix}$$

Given that 8 is an eigenvalue of **M**

(a) determine an eigenvector corresponding to the eigenvalue 8

(2)

(b) Determine the other two eigenvalues of **M**.

(3)

(c) Hence find an orthogonal matrix **P** and a diagonal matrix **D** such that $\mathbf{P}^{\mathrm{T}}\mathbf{M}\mathbf{P} = \mathbf{D}$

(4)

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Question 5 continued

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3.

$$\mathbf{M} = \begin{pmatrix} 3 & -4 & k \\ 1 & -2 & k \\ 1 & -5 & 5 \end{pmatrix} \quad \text{where } k \text{ is a constant}$$

Given that 3 is an eigenvalue of **M**,

(a) find the value of k.

(3)

(b) Hence find the other two eigenvalues of **M**.

(3)

(c) Find a normalised eigenvector corresponding to the eigenvalue 3

(3)

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Question 3 continued

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6.

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & a \end{pmatrix} \quad a \neq 1$$

(a) Find $\mathbf{A}^{-1}$ in terms of a.

(4)

$$\mathbf{B} = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{pmatrix}$$

The straight line l_1 is mapped onto the straight line l_2 by the transformation represented by the matrix **B**.

The equation of l_2 is

$$\left(\mathbf{r} - (12\mathbf{i} + 4\mathbf{j} + 6\mathbf{k})\right) \times (-6\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) = \mathbf{0}$$

(b) Find a vector equation for the line l_1

(4)

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Question 6 continued

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